

A Map Algebra Approach to Analyzing Spatiotemporal Data

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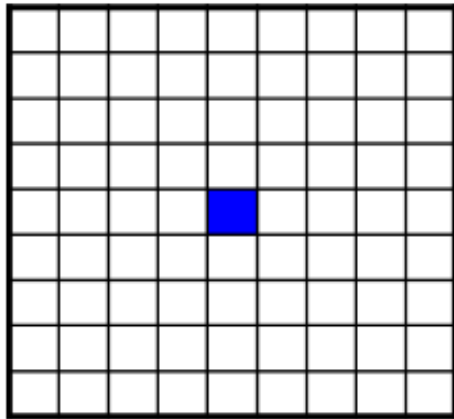
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Outline

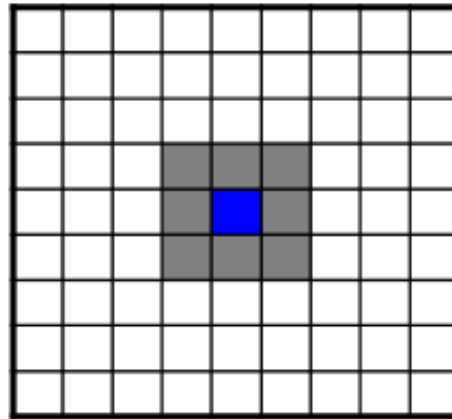
- Map algebra and its extensions
- Nature of map algebra
- Extension to time series rasters

The Original Raster Map Algebra

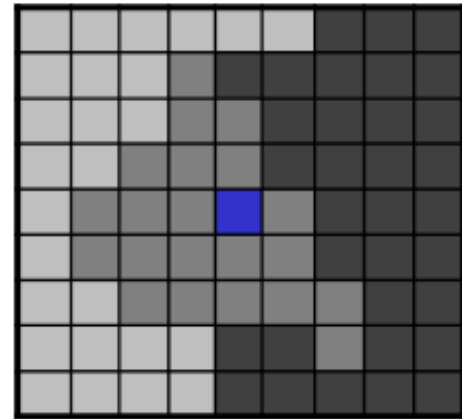
- Tomlin (1990, 2012) organizes raster analysis operations as **local**, **focal**, and **zonal** according to the *spatial scope* of those operations



Local



Focal



Zonal

Major Extensions to MA

- 2D (pixels/cells) → 3D (voxels/cubes)
 - Scott (1999) and Neteler (2004)
- Scalar raster → vector raster
 - Li and Hodgson (2004) and Wang and Pullar (2005)
- Feature-based
 - French and Li (2010)
- Flow network (raster & vector)
 - Tarboton and Baker (2008)
 - She & Li (2016)
- Time series of rasters
 - Mennis et al. (2005) and Mennis (2010)

The Nature of Map Algebra

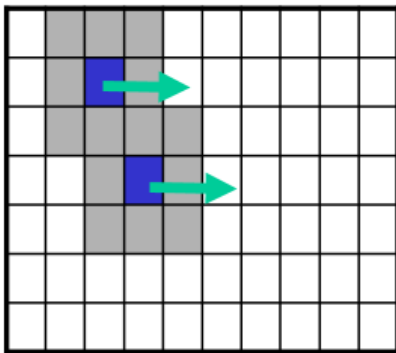
- Simple but powerful approach
 - Primarily way of analyzing raster data
 - Implemented in commercial /open source GIS software and cloud-based geospatial analysis platform
- What's the nature of map algebra?
 - What kind of computational instrument does MA provide?

Neighborhoods and Zones

- “A neighborhood is a set of locations at specified cartographic distances and/or directions from a given location” (Tomlin, 2012)
- “A zone is the set of data pertaining to a specific geographic conditions. The cartographic form of a zone can be large or small and in one piece or in a number of disconnected fragments.”
(Tomlin, 2012)

Zones Are Stored Neighborhoods

- A zone is a neighborhood where all the cells in the zone share exactly the same neighborhood
- The zone raster is a map of neighborhoods
- Zones don't overlap in space



1	1	1	1	1
1	3	3	3	1
1	1	2	2	2
1	1	2	3	2
1	3	3	3	2

The Nature of Map Algebra

- Perform an operation within a cell's neighborhood on a raster
- *Iteration*
 - Perform the operation at each cell (spatialization)
 - Iterate through the cells on a raster
- *Neighborhood*
 - Define the cells related to a cell
 - Represent a certain relationship between a cell and its neighborhood cells
- *Operation*
 - Data manipulations performed on neighborhood cells

Neighborhood

- Neighborhood(cell, otherArgs)
 - cell—currently processed cell
 - otherArgs—additional parameters used to define neighborhood
 - Returns a set of cells called the neighborhood of the cell
- Represents a certain relationship between a cell and its neighborhood cell(s)
 - Link location based relationships
- Examples
 - AdjacentNeighborhood(cell, kernel)
 - DistanceDirectionNeighborhood(cell, distance, direction)
 - NearestNeighbor(cell, featuresRaster)
 - Watershed(cell, flowDirectionRaster)
 - Viewshed(cell, visibleDistance, offset, ...)

Data Manipulation Operation

- Operation(cell, valueRasters, otherArgs)
 - cell—currently processing cell
 - valueRasters--A set of rasters from which values are retrieved
 - otherArgs--Additional parameters used in data manipulation
- Major steps
 - Get the neighborhood cells from Neighborhood() function
 - Retrieve values from valueRasters at neighborhoodCells and/or cell
 - Perform data manipulation
 - location (neighborhoodCells and/or cell)
 - values (at neighborhoodCells and/or cell) retrieved from valueRasters
 - Return a value or a set of values

Link Data by Location

- Link data (raster values) at neighborhood cells and/or cell
 - $v = f(\text{cell}, \text{rasters})$
 - Cell and rasters may have different size
- Link data at the cell and at its neighborhood cells
 - Link data through the link in location
 - Link in location defined Neighborhood

Cartographic Modeling “Operations”

- “Local operations” —use the cell value at the same location
 - Neighborhood(cell)
 - Returns the cell
- “Zonal operations” —use the cell values within the same zone
 - Neighborhood(cell, zoneRaster)
 - Get the value of the cell on zoneRaster
 - Returns the cells with the same value on zoneRaster as neighborhood cells
 - Neighborhood is defined and stored in zoneRaster
- “Focal operations” —use the cell values bear a certain distance and/or direction
 - Neighborhood(cell, distance, direction)
 - Returns the cells bearing certain distance and direction from the cell as neighborhood cells

The Nature of Map Algebra

- What does MA offer?
 - A form of convolution?
 - Iteration
 - Neighborhoods
- Reveal emergent spatial patterns/forms by convolution
 - Spatial consequences/effects from local relationships represented by neighborhoods
- Geographical convolution
 - Neighborhood defined in geographical space
 - Convolution on multiple attributes (local neighborhood)
 - Convolution on irregular neighborhoods (watershed) and different neighborhood at different cells
- A computational instrument helps see what we cannot see
 - Explore local relationships and emergent forms

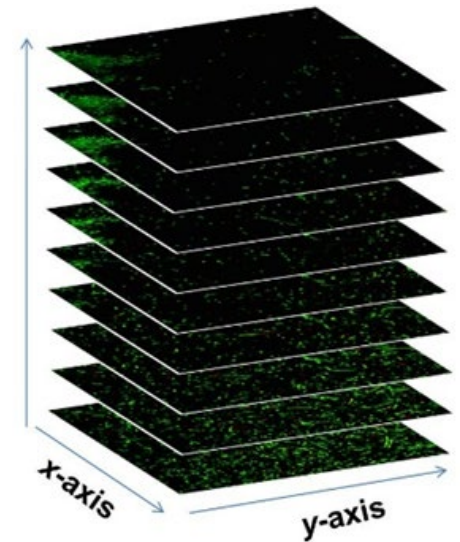
Drainage Networks As an Emergent Form

- Watershed() as the neighborhood
- valueRaster = 1
- Sum the values within a cell's watershed neighborhood



Map Algebra for Time Series of Rasters

- Perform an operation within a cell's neighborhood on a time series of rasters
- *Iteration in space and time*
 - Perform the operation at each cell and time
 - Iterate through the cells in space and time
- *Neighborhood in space and time*
 - Define the cells related to a cell in space and time
 - Represent a certain relationship between a cell and its neighborhood cells in space and time
- *Operation*
 - Data manipulations performed on neighborhood cells



Spatiotemporal Neighborhoods

- Neighborhood(*tsCell*, *otherArgs*)
 - *tsCell*—currently processed cell in time and space
 - *otherArgs*—additional parameters used to define neighborhood
 - Returns a set of cells
- Represents a certain relationship between a *tsCell* and its neighborhood *tsCell(s)*
 - Link location and time based relationships
- Examples
 - AdjacentNeighborhood(*tsCell*, *tsKernel*)

0	1	0
1	1	1
0	1	0

t-1

1	1	1
1	1	1
1	1	1

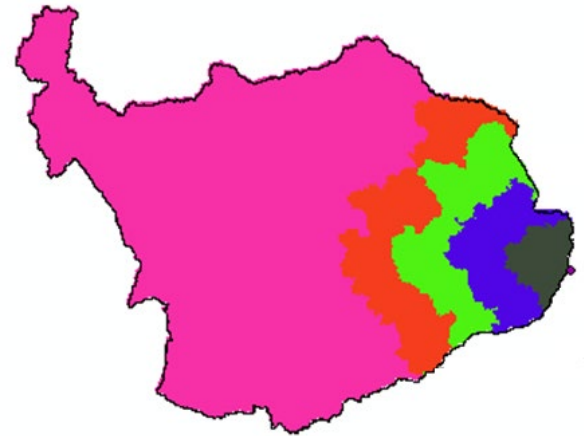
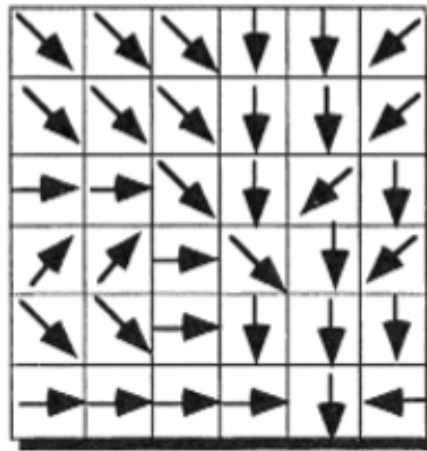
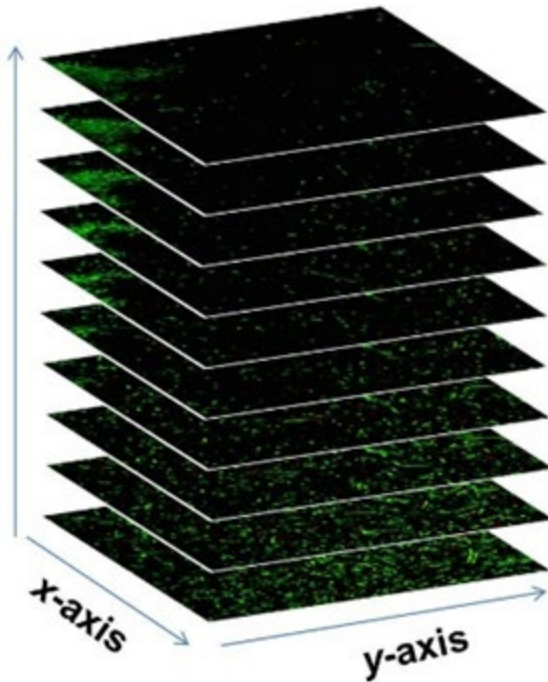
t

0	1	0
1	1	1
0	1	0

t+1

Watershed as Neighborhood

- `Watershed(cell, flowDirectionRaster, flowSpeed)`
- It takes time for the water in a cell's watershed to reach the cell
- Inflow at each cell's the time of concentration



Spatiotemporal Neighborhoods

- Define neighborhood based the interactions between spatial and temporal component neighborhoods
 - Local spatiotemporal neighborhoods (1)
 - Zonal spatiotemporal neighborhoods (2, 3, 4)
 - Focal spatiotemporal neighborhoods (5, 6, 7, 8, 9)

		Time		
		local	zonal	focal
Space	local	1	2	5
	zonal	3	4	6
	focal	7	8	9

Data Manipulation Operation

- Operation(tsCell, tsRasters, otherArgs)
 - tsCell—currently processing cell in time and space
 - tsRasters--A set of time series rasters from which values are retrieved
 - otherArgs--Additional parameters used in data manipulation
- Major steps
 - Get the neighborhood tsCells from Neighborhood() function
 - Retrieve values from tsRasters at neighborhood tsCells and/or tsCell
 - Perform data manipulation
 - Location of neighborhood tsCells and/or tsCell)
 - Time of neighborhood tsCells and/or tsCell)
 - Values at neighborhood tsCells and/or tsCell retrieved from tsRasters
 - Return a value or a set of values

Link Data by Location and Time

- Link data (tsRaster values) at neighborhood tsCells and/or tsCell
 - $v = f(\text{tsCell}, \text{tsRasters})$
 - tsCell and tsRasters may have different spatial and temporal resolutions
- Link data at the tsCell and at its neighborhood tsCells
 - Link data through the link in location and time
 - Link in location and time is defined by Neighborhood
- Time
 - Local vs absolute
 - Circular (days, years)
 - Relationship between time and attribute

Conclusions

- Zones are special neighborhoods
- Map algebra as a computational instrument for geographical convolution
- Extension to analyze time series of rasters
- Future work
 - Vector data model and spatiotemporal vector data
 - Implementation

Questions?

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