A Map Algebra Approach to Analyzing Spatiotemporal Data

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Outline

• Map algebra and its extensions
• Nature of map algebra
• Extension to time series rasters
Tomlin (1990, 2012) organizes raster analysis operations as **local**, **focal**, and **zonal** according to the *spatial scope* of those operations.
Major Extensions to MA

• 2D (pixels/cells) → 3D (voxels/cubes)
• Scalar raster → vector raster
• Feature-based
  – French and Li (2010)
• Flow network (raster & vector)
  – Tarboton and Baker (2008)
  – She & Li (2016)
• Time series of rasters
  – Mennis et al. (2005) and Mennis (2010)
The Nature of Map Algebra

• Simple but powerful approach
  – Primarily way of analyzing raster data
  – Implemented in commercial / open source GIS software and cloud-based geospatial analysis platform

• What’s the nature of map algebra?
  – What kind of computational instrument does MA provide?
Neighborhoods and Zones

• “A neighborhood is a set of locations at specified cartographic distances and/or directions from a given location” (Tomlin, 2012)

• “A zone is the set of data pertaining to a specific geographic conditions. The cartographic form of a zone can be large or small and in one piece or in a number of disconnected fragments.” (Tomlin, 2012)
Zones Are Stored Neighborhoods

- A zone is a neighborhood where all the cells in the zone share exactly the same neighborhood
- The zone raster is a map of neighborhoods
- Zones don’t overlap in space
The Nature of Map Algebra

• Perform an operation within a cell’s neighborhood on a raster

• **Iteration**
  – Perform the operation at each cell (spatialization)
  – Iterate through the cells on a raster

• **Neighborhood**
  – Define the cells related to a cell
  – Represent a certain relationship between a cell and its neighborhood cells

• **Operation**
  – Data manipulations performed on neighborhood cells
Neighborhood

• Neighborhood(cell, otherArgs)
  – cell—currently processed cell
  – otherArgs—additional parameters used to define neighborhood
  – Returns a set of cells called the neighborhood of the cell

• Represents a certain relationship between a cell and its neighborhood cell(s)
  – Link location based relationships

• Examples
  – AdjacentNeighborhood(cell, kernel)
  – DistanceDirectionNeighborhood(cell, distance, direction)
  – NearestNeighbor(cell, featuresRaster)
  – Watershed(cell, flowDirectionRaster)
  – Viewshed(cell, visibleDistance, offset, ...)
Data Manipulation Operation

• Operation(cell, valueRasters, otherArgs)
  – cell—currently processing cell
  – valueRasters—A set of rasters from which values are retrieved
  – otherArgs—Additional parameters used in data manipulation

• Major steps
  – Get the neighborhood cells from Neighborhood() function
  – Retrieve values from valueRasters at neighborhoodCells and/or cell
  – Perform data manipulation
    • location (neighborhoodCells and/or cell)
    • values (at neighborhoodCells and/or cell) retrieved from valueRasters
  – Return a value or a set of values
Link Data by Location

• Link data (raster values) at neighborhood cells and/or cell
  – \( v = f(\text{cell, rasters}) \)
  – Cell and rasters may have different size

• Link data at the cell and at its neighborhood cells
  – Link data through the link in location
  – Link in location defined Neighborhood
Cartographic Modeling “Operations”

- **“Local operations”**—use the cell value at the same location
  - Neighborhood(cell)
    - Returns the cell

- **“Zonal operations”**—use the cell values within the same zone
  - Neighborhood(cell, zoneRaster)
    - Get the value of the cell on zoneRaster
    - Returns the cells with the same value on zoneRaster as neighborhood cells
  - Neighborhood is defined and stored in zoneRaster

- **“Focal operations”**—use the cell values bear a certain distance and/or direction
  - Neighborhood(cell, distance, direction)
    - Returns the cells bearing certain distance and direction from the cell as neighborhood cells
The Nature of Map Algebra

• What does MA offer?
  – A form of convolution?
  – Iteration
  – Neighborhoods

• Reveal emergent spatial patterns/forms by convolution
  – Spatial consequences/effects from local relationships represented by neighborhoods

• Geographical convolution
  – Neighborhood defined in geographical space
  – Convolution on multiple attributes (local neighborhood)
  – Convolution on irregular neighborhoods (watershed) and different neighborhood at different cells

• A computational instrument helps see what we cannot see
  – Explore local relationships and emergent forms
Drainage Networks As an Emergent Form

- Watershed() as the neighborhood
- valueRaster = 1
- Sum the values within a cell’s watershed neighborhood
Map Algebra for Time Series of Rasters

• Perform an operation within a cell’s neighborhood on a time series of rasters

• *Iteration in space and time*
  – Perform the operation at each cell and time
  – Iterate through the cells in space and time

• *Neighborhood in space and time*
  – Define the cells related to a cell in space and time
  – Represent a certain relationship between a cell and its neighborhood cells in space and time

• *Operation*
  – Data manipulations performed on neighborhood cells
Spatiotemporal Neighborhoods

- Neighborhood(tsCell, otherArgs)
  - tsCell—currently processed cell in time and space
  - otherArgs—additional parameters used to define neighborhood
  - Returns a set of cells

- Represents a certain relationship between a tsCell and its neighborhood tsCell(s)
  - Link location and time based relationships

- Examples
  - AdjacentNeighborhood(tsCell, tsKernel)

```
0 1 0
1 1 1
0 1 0

1 1 1
1 1 1
1 1 1

0 1 0
1 1 1
0 1 0
```

```
0 1 0
1 1 1
0 1 0

1 1 1
1 1 1
0 1 0
```

```
0 1 0
1 1 1
0 1 0
```
Watershed as Neighborhood

- Watershed(cell, flowDirectionRaster, flowSpeed)
- It takes time for the water in a cell’s watershed to reach the cell
- Inflow at each cell’s the time of concentration
Spatiotemporal Neighborhoods

- Define neighborhood based on interactions between spatial and temporal component neighborhoods
  - Local spatiotemporal neighborhoods (1)
  - Zonal spatiotemporal neighborhoods (2, 3, 4)
  - Focal spatiotemporal neighborhoods (5, 6, 7, 8, 9)

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Data Manipulation Operation

• Operation(tsCell, tsRasters, otherArgs)
  – tsCell—currently processing cell in time and space
  – tsRasters--A set of time series rasters from which values are retrieved
  – otherArgs--Additional parameters used in data manipulation

• Major steps
  – Get the neighborhood tsCells from Neighborhood() function
  – Retrieve values from tsRasters at neighborhood tsCells and/or tsCell
  – Perform data manipulation
    • Location of neighborhood tsCells and/or tsCell)
    • Time of neighborhood tsCells and/or tsCell)
    • Values at neighborhood tsCells and/or tsCell retrieved from tsRasters
  – Return a value or a set of values
Link Data by Location and Time

• Link data (tsRaster values) at neighborhood tsCells and/or tsCell
  – \( v = f(tsCell, tsRasters) \)
  – tsCell and tsRasters may have different spatial and temporal resolutions

• Link data at the tsCell and at its neighborhood tsCells
  – Link data through the link in location and time
  – Link in location and time is defined by Neighborhood

• Time
  – Local vs absolute
  – Circular (days, years)
  – Relationship between time and attribute
Conclusions

• Zones are special neighborhoods
• Map algebra as a computational instrument for geographical convolution
• Extension to analyze time series of rasters
• Future work
  – Vector data model and spatiotemporal vector data
  – Implementation
Questions?

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